UDC 666.1.031.13:532.5

METHODOLOGY FOR ADAPTATION OF A CONJUGATE MATHEMATICAL MODEL OF HYDRODYNAMICS AND HEAT EXCHANGE IN A GLASS-MELTING FURNACE WITH HORSESHOE-SHAPED FLAME DIRECTION

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Translated from *Steklo i Keramika*, No. 12, pp. 7 – 11, December, 2006.

The sequence for adapting a conjugate mathematical model describing external heat exchange in the work space and heat exchange and hydrodynamics of the melt in a glass-melting furnace melting tank with horseshoe-shaped flame direction was examined. It was shown that it is sufficient to use the experimental dependences of the air heating temperature and temperature of the glass melt at the outlet of the melting tank on the furnace efficiency for adapting the model when the basic modeling boundary conditions have been correctly assigned. Equations were obtained that establish the correlation of the specific heat rate and maximum melting temperature and the specific output of glass melt, and they can be used in constructing an algorithm for controlling the thermal work of a glass-melting furnace of the design studied.

In the current conditions of development of the glass industry, mathematical modeling of glass-melting furnaces should be considered an integral part of design methodology [1]. Mathematical models that are sufficiently complete and adequate can be used to not only evaluate the reliability of the basic characteristics of furnaces but also to optimize them and to predict their behavior with consideration of the probable nature of many operating parameters of real objects. Predicting the output characteristics of a designed object (output, fuel consumption, melting temperature, etc.) in the design stage is due to the necessity of ensuring the requirements for the quality of the glass melt, thermal efficiency of the furnace, and operating time between servicings. It makes it possible to evaluate the correspondence of the furnace design to the performance loads and purposefully select the optimum operating parameters.

In the general case, the detail and accuracy of the mathematical model should correspond to the given quality of the object. Superfluous details in the model causes an unjustified increase in computer time costs and insufficient detail leads to an incorrect result. On the whole, the basic requirements for mathematical models can be reduced to goodness of fit, accuracy, and economy. Economy (computer efficiency) is characterized by the costs for computer time and memory of the computer system. Goodness of fit is the reflection of the selected properties of the object by the model with a given accuracy. Accuracy means the degree of correspondence of the evaluations of similar properties of the object and model.

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The goodness of fit of the model to the sample is always limited and is a function of the purpose of modeling. Any model does not take into account some properties of the original and for this reason is an abstraction of it. The goodness of fit of the model is established by testing the basic laws of the object region (for example, laws of conservation) and comparing the results of modeling of particular variants with the known solutions for these variants. Within the framework of the principles of reasonable sufficiency and balance of accuracy, the evaluation of the goodness of fit of a mathematical model in the first stage consists of establishing the physicality of the results of the calculations [2, 3]. Here physicality means the qualitative correspondence of the mechanisms revealed in modeling developed in the object region to the representations or available experimental data. On the quantitative level, the degree of agreement of the calculated quantities with the real parameters of the full-scale object is determined by the modeling goals.

We can distinguish two limiting cases of the use of mathematical modeling of glass-melting furnaces. In the first one, the effect of some design (process) parameters of the aggregate on its operation must be established. It is evident that modeling in this case will boil down to executing a series of comparative calculations, and high accuracy of correspondence of the calculated data with the real values is not required here [4 – 6]. It is only important for the effect of the investigated parameters on the evolution of heat exchange and hydrodynamic processes to be reflected in the model. In other words, establishing the physicality is the task of evalu-

ating the goodness of fit of a mathematical model in the given case, and for this reason, this evaluation can frequently only be conducted without involving experimental data.

In the second limiting case, the mathematical model is either used as a constituent part of the system for automatic control of the object or for revealing "bottlenecks" in this object; for example, for determining fuel consumption, the maximum temperate of the protective lining, glass-melt temperature at the furnace outlet, etc. Since the working characteristics of the design elements are taken into consideration in the calculation, in the given case, higher requirements are imposed on the evaluation of the goodness of fit of the mathematical model and they can only be satisfied if special experiments are conducted. Adaptation of a mathematical model not only implies refining the initial modeling conditions, but also quantitatively evaluating the goodness of fit of the basic integral indexes of operation of a furnace of analogous design. Incorporating a furnace efficiency variable is the basic condition for executing this procedure.

As an illustration of the above, consider the sequence of adaptation of a conjugate mathematical model of hydrodynamics and heat exchange in a glass-melting furnace with horseshoe-shaped flame direction which is completely characterized in [2, 3]. We will use the operating indexes of glass-melting furnace No. 1 at Orekhovo-Zuevo Glass Company as the experimental data [1]. The results of mathematical modeling were used in designing this furnace, and the difference between it and the model only consisted of the area of the melting tank ($F_t = 102.4\ \text{m}^2$) and the lower heat value of natural gas ($Q_l^r = 33.446\ \text{MJ/m}^3$). In modeling these values were set equal to 115.77 m^2 and 35.042 MJ/m^3 in [2]. The experimental data were obtained for the furnace output range of 115.0 – 241.1 tons/day.

Consider the furnace operating parameters used for evaluating the goodness of fit of the modeling results. The control-measurement complex of a modern glass-melting furnace allows determining its output in fused glass melt P (tons/day), fuel consumption B (m^3/day), temperature of the inside of the roof, including the maximum value $t_{\text{r.m}}$, air heating temperature in the regenerator t_a, and weight-average temperature of the glass melt at the furnace outlet $\bar{t}_g$. These parameters can be used to calculate the specific fuel consumption B_{sp} or the specific heat flow q_{sp}, as well as the specific output of glass melt from 1 m^2 of melting tank surface area P_{sp} (ton/(m^2 · day)). It is thus possible to determine the dependences that are integral characteristics of the thermal efficiency of a glass-melting furnace:

$$B_{\text{sp}} = f(P),\ \text{m}^3/\text{ton}; \tag{1}$$

$$q_{\text{sp}} = f(P_{\text{sp}}),\ \text{kJ/kg}. \tag{2}$$

Equation (1) can be directly used in the algorithm for controlling the thermal work of the furnace. Equation (2) is the most universal characteristic of operation of a furnace generally accepted in world practice. Using it allows not only comparing the operating indexes of different furnaces of the same design type but also the results of modeling with experimental data obtained on samples with values of F_t and Q_l^r that differ from the model. On the whole, functions (1) and (2) can be used for evaluating the goodness of fit of the mathematical model to the industrial sample.

The convergence of the numerical values of Eqs. (1) and to a greater degree (2) for the model and sample is an important but not the only condition for testing the goodness of fit of the results of the calculation. We know that in practical glass making, controlling the thermal work of the furnaces is based on some "arbitrary" indexes on which the furnace output and glass quality are to some degree dependent. They include not only the fuel (or heat) consumption but also the temperature distribution in the working space of the furnace and its (temperature) level [7]. It is customarily assumed that the conditions required for the successful occurrence of the manufacturing process can be created by correctly assigning the level and position of the temperature maximum.

The temperature in the furnace is one of the most complex control parameters. This is first due to the complexity of measuring the real temperature and limiting the number of control points, which does not give a sufficiently complete representation of the temperature field in the furnace. We know that the gas medium in the working space of high-temperature furnaces is characterized by a nonuniform temperature field. For this reason, the term "furnace temperature" is arbitrary in character, and the notion of "effective temperature" is used for characterizing the thermal state of the working space. Second, this is related to the low inertness of the object regulated and the mutual effect of the temperatures in the furnace and the controlling action — fuel or power consumption.

To increase the stability of operation of the control system, slight "coarsening" of the primary signal is useful. This implies selecting a more inert control object. The temperature of the inner surface of the working space lining, preferably the furnace roof, can serve as such an object. Assigning the temperature curve and especially its maximum based on the temperature of the inside of the roof produces several positive results. First, more accurate measurement of this temperature by mounting the case of the thermocouple flush with the inner surface of the lining. Second, objective control of the maximum roof temperature is ensured, which prevents accidents related to its overheating. Third, the roof temperature is more inert to brief deviations in the furnace operating regulations. And finally, and most important, in certain conditions, the maximum roof temperature is satisfactorily correlated with other furnace thermotechnical and performance parameters. The maximum value of the temperature of the inner surface of the roof and its position over the length of the furnace can thus be included among the basic parameters in testing the goodness of fit of the mathematical model to the industrial sample [7].

Elaboration of a mathematical model of a glass-melting furnace that satisfactorily fits the sample in a wide output

range (for example, 100 – 300 tons/day) presupposes correct definition of some of the most important initial conditions of modeling: input of heat with the heated air and loss of heat with the working glass-melt stream. At constant output (P = const), these items can be determined from calculating the heat balance of the melting part of the furnace. For variable furnace output, formalization of the dependences $t_a = f(P)$ and $\bar{t}_{g.m} = f(P)$ implies adaptation of the model using the experimental data. The necessity of executing this procedure can be seen with the data in Fig. 1, where the calculated values for different conditions of formalizing $t_a = f(P)$ and $\bar{t}_{g.m} = f(P)$ are shown together with the experimental dependence of B_{sp}^{ex}. We note that the experimental data are approximated by the equation (standard error of approximation $R^2 = 0.9778$):

$$B_{sp}^{ex} = 239 - 0.6346P + 0.0009P^2, \text{ m}^3/\text{ton}.$$

The heat expended for glass formation Q_1' and heating the glass melt in the tank Q_1'' to 1500°C at different furnace output was calculated with the equations:

$$Q_1' = -0.240668 + 11.280498P, \text{ kJ};$$

$$Q_1'' = -1.393369 + 27.497431P, \text{ kJ}.$$

The method of distribution of Q_1' and Q_1'' in the tank is given in [2].

If the mathematical model is restricted by the first limiting case of its application, then the simplest method of defining the initial conditions with $t_a = f(P)$ and $\bar{t}_{g.m} = f(P)$ is possible. It consists of the fact that these parameters are defined in the form of fixed quantities whose concrete values should be correlated (from an experiment) with the furnace output used. If P = var and varies in a relatively wide range, then implementation of this variant of defining the initial conditions, for example, t_a = 1200°C and $\bar{t}_{g.m}$ = 1300°C (see Fig. 1, curve *2*) will lead to an important difference in the calculated and experimental data, both qualitatively and quantitatively.

The source of the calculation error (maximum deviation of 9.45%) is primarily due to the fact that at low furnace output, the amount of heat introduced in the regenerator by the products of combustion cannot ensure the given level of heating of the air in combustion. In addition, it is also predetermined by the iteration method of determining the fuel consumption at which the calculations terminated when the weight-average temperature of the glass melt in the neck attained the assigned value.

Another possible method of defining the air heating temperature that to some degree correlates t_a with furnace output and fuel consumption consists of using a relation of the type

$$t_a = t_d - \Delta t.$$

The temperature of the outgoing fuel combustion products t_d is determined by calculation. Quantity Δt (experimental) can be varied within the limits of 150 – 300°C. The variant of the calculation for Δt = 300°C and $\bar{t}_{g.m}$ = 1300°C is shown in Fig. 1 (see curve *3*). The results of the calculation indicate that the character of the dependence $B_{sp} = f(P)$ did not undergo any major changes in comparison to the preceding variant (see curve *2*).

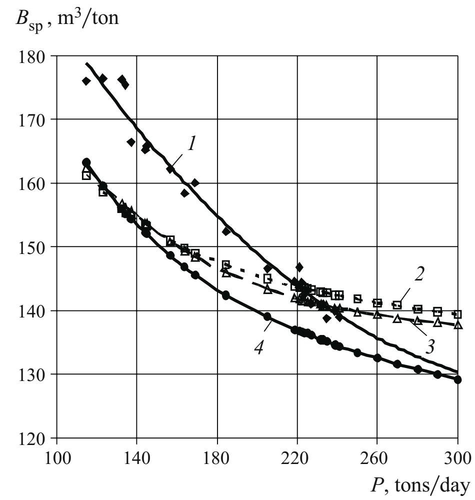

Fig. 1. Effect of furnace output on fuel rate: *1* and *2* – *4*) experimental and calculated data.

On the whole, we note that the examined methods of assigning the initial conditions with t_a and $\bar{t}_{g.m}$ can be used for solving a number of problems related to the first limiting case of use of mathematical modeling. The value of the calculation error and character of the change in $B_{sp} = f(P)$ sufficiently correctly reflects the real operating indexes of glass-melting furnaces. At the same time, going to problems of the second limiting case requires more complete both quantitative and qualitative correspondence of the calculated and experimental data. Attaining this goal is seen in adaptation of the model using the experimental dependences $t_a = f(P)$ and $\bar{t}_{g.m} = f(P)$.

The analysis of the results of operating an industrial sample showed that the temperature of heating the air for combustion and the weight-average temperature of the glass melt at the furnace outlet are in certain dependences on its output (Fig. 2). These dependences can be sufficiently correctly approximated by second-degree polynomials. The equations are as follows for t_a ($R^2 = 0.9833$) and $\bar{t}_{g.m}$ ($R^2 = 0.9937$):

$$t_a = 1009.5 + 124.4P - 3.8933P^2, \text{°C}; \quad (3)$$

$$\bar{t}_{g.m} = 1414.6 + 0.3299P - 0.0003P^2, \text{°C}. \quad (4)$$

The use of Eqs. (3) and (4) as the initial modeling conditions allows obtaining a new calculation expression for function $B_{sp} = f(P)$ (see Fig. 1, curve *4*):

$$B_{sp} = 108.223 + \frac{6317.617}{P}, \text{ m}^3/\text{ton}. \quad (5)$$

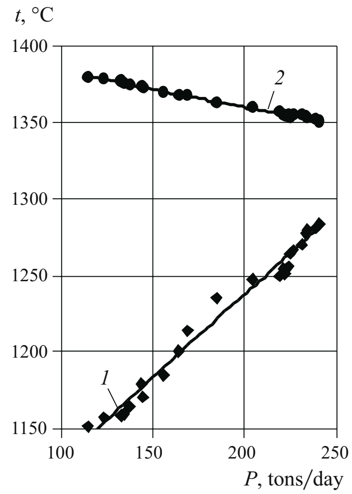

Fig. 2. Experimental dependence of the air heating temperature (*1*) and glass melt temperature at the neck outlet (*2*) on furnace output.

The character of function (5), adapted with experimental data, more accurately reflects the change in the fuel value as a function of the furnace output (see Fig. 1, curve *1*). The maximum calculation error is 8.68% and the average deviation is equal to 6%.

We note that curves *1* and *4* (see Fig. 1) were obtained for different values of the melting tank surface area and lowest operating efficiency of the fuel. For this reason, a correct comparison of the calculated and experimental data implies their representation in the form of dependences described by equations of the type of (2). The data in Fig. 3*a* indicate that in generalized form, the results of calculating $q_{sp} = f(P_{sp})$ is satisfactorily correlated with the experimental data. The higher the specific output of the furnace, the better the convergence of the results of the calculation and the experimental data. This is because the initial conditions on heat losses through furnace structural elements were assigned for furnace output of 300 tons/day [2]. The average deviation of the calculation is 3.86%, which indicates the good fit of the mathematical model to the real process. The calculated curve of function $q_{sp} = f(P_{sp})$ can be described by the following equation ($R^2 = 0.9911$):

$$q_{sp} = 6668.3 - 1316.3P_{sp} + 181.98P_{sp}^2, \text{ kJ/kg.} \quad (6)$$

Adapting the mathematical model with the initial conditions of assignment of t_a and $\bar{t}_{g.m}$ thus allows obtaining the equation correlating the heat value with the specific furnace output, which indicates the suitability of the model for solving problems related to the second limiting case. Equation (6) can be used in the algorithm of the equation for the thermal work of furnaces of the given design in a wide output range.

The dependence of the maximum roof temperature on the specific furnace output is shown in Fig. 3*b*. The calculated values were obtained with consideration of adaptation of the mathematical model using the experimental data in

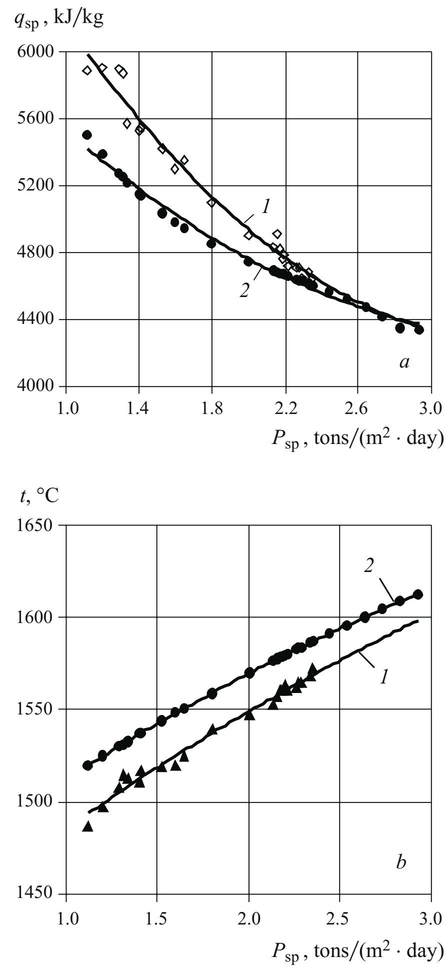

Fig. 3. Heat value (*a*) of the maximum roof temperature (*b*) on unit furnace output: *1* and *2*) experimental and calculated data.

Fig. 2 and approximated equations (3) and (4). Processing the results of the calculation showed that function $t_{r.m} = f(P_{sp})$ can be described by the expression ($R^2 = 0.9978$):

$$t_{r.m} = 1443.4 - 74.557P_{sp} + 5.67593P_{sp}^2. \quad (7)$$

The data in Fig. 3*b* indicate that the results of the calculation correlate satisfactorily with the experimental values both qualitatively and quantitatively. The average error of calculating the maximum roof temperature does not exceed 1.4%.

The character of the change in function $t_{r.m} = f(P_{sp})$ corresponds to the developed concepts not only concerning the level of the maximum roof temperature but also the change in it as a function of the specific output of modern glass-melting furnaces with horseshoe-shaped flame direction. For example, for P_{sp} = 1.0, 1.5, 2.0, 2.5, and 3.0 tons/(m^2 · day), with Eq. (7) we obtain $t_{r.m}$ = 1512, 1542, 1569, 1594, and 1615°C. Two conclusions derive from the reported data which indicate the goodness of fit of Eq. (7) to glass-melting practice. The first is that the increase in the specific furnace output will outstrip (in percentage) the required increase in

the melting temperature. The second is that for a flame furnace (without using additional electric heating), the specific glass-melt output of 2.5 tons/($m^2 \cdot$ day) will be the limiting value with respect to the temperature of its upper structure.

In conclusion, it should be noted that the conjugated mathematical model of a glass-melting furnace with horseshoe-shaped flame direction developed [2, 3] satisfactorily reflects the thermophysical processes that characterize operation of an industrial sample. In correct assignment of the basic boundary conditions of modeling (set of initial and boundary conditions), it is sufficient to use the experimental dependences of the air heating temperature and glass-melt temperature at the melting tank outlet on the furnace output for adapting the model.

Equations (5) – (7) can be used in constructing an algorithm for control of the thermal work of a glass-melting furnace of the investigated design.

The development of systems for controlling the thermal work of glass-melting furnaces is correlated with the creation of multilevel algorithms. Despite the attractiveness of the roof temperature as object of regulation, this parameter does not fully reflect all of the complexity of the processes that take place in the furnace, and is also not a direct characteristic of any aspects of the quality of the glass melt. Development of numerical modeling methods will allow making the control algorithm more complex, more reliable, and better fit the furnace operating conditions. The possibility of a combined solution to external and internal heat-exchange problems will form the basis for passing to control of the thermal work of the furnace through regulation (stabilization) of the temperature of the glass melt in the basic zones of the melting tank.

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